Proceedings of the 2016 3rd Conference on Control and Fault-Tolerant Systems (SysTol), Barcelona, Spain, Sept. 7-9, 2016

Model-based prognosis algorithms with uncertainty propagation: application to fatigue crack growth

Elinirina Irena Robinson[1], Julien Marzat[1], and Tarek Raïssi[2]

Abstract— **In this paper, deterministic and stochastic nonlinear prognosis methods that take uncertainty propagation into account are evaluated. More specifically, a deterministic method using interval techniques and two stochastic methods based on Bayesian filtering, namely extended Kalman filter and particle filter, are considered. The three algorithms are compared with reference to a classical benchmark which is a crack growth analysis, however they can be extended to other applications as well. The advantages and drawbacks of each approach are studied through different prognosis metrics such as accuracy, precision and timeliness. Based on these numerical simulations, the results show that deterministic methods for prognosis are suitable to manage bounded uncertainty.**

I. INTRODUCTION

Critical systems such as an aircraft or a spacecraft are made of complex components whose malfunction and failure could have unacceptable impacts on the users safety, the mission success and the costs related to maintenance operations. In order to avoid catastrophic scenarios, diagnosis and prognosis modules are incorporated to these systems. Diagnosis is defined as the detection, isolation and identification of a failure that has occurred in the system, whereas prognosis aims at estimating the remaining useful life (RUL) of a system once the diagnosis step has been done.

There are various prognosis approaches, but the most common classification divides them into three main categories [1]. The first one gathers the knowledge-based approaches [2], where the degradation rules have been developed and refined by experts based on historical and empirical failure data. The second one includes data-driven approaches [3], which extract features from operating data such as current, temperature, or vibration signals. They mainly use statistical and machine learning techniques to track, approximate and forecast the evolution of the degradation state. The third category focuses on the model-based prognosis approaches [4] through the use of a dynamic mathematical model of the process being monitored. Each of these approaches has its advantages and drawbacks, and the choice of the method to use depends on the application domain and the information available about the system. The knowledge-based approaches are easy to implement, but frequent updates are needed as new forms of faults that are not yet listed can occur. Data-driven approaches have the ability to transform high-dimensional noisy data into lower-dimensional information for prognosis decisions. However, they are highly-dependent on the quantity and quality of operational data and therefore require a significant storage space. Model-based prognosis approaches need an accurate degradation model, which can be difficult to obtain in most cases. However they have the potential to outperform the two other approaches. Indeed, the ability to incorporate physical knowledge of the system is the main advantage of the model-based approaches. Moreover, model adaptation to a system degradation is another advantage because it helps to keep the prognosis accuracy at a required level if the of the system degradation is improved. Therefore in this paper, the focus is entirely placed on model-based prognosis techniques.

In the literature, various model-based prognosis approaches have been developed [5], but the uncertainty management problem has only recently been addressed ([6], [7]), whereas it is a key aspect of prognosis [8]. Indeed, since the prediction of the RUL of a degrading system is accomplished in the absence of future measurements, it is unavoidably affected by coarse uncertainty. The objective of uncertainty management is to determine the sources of uncertainty and propagate them to get the probability density function (pdf) of the predicted RUL. There are mainly three major sources of uncertainty: modeling uncertainty, sensor measurement uncertainty and operational uncertainties.

In this context, the aim of the paper is to present and compare the ability of three model-based prognosis methods to deal with uncertainties in a nonlinear framework. Usually, in order to take uncertainties into account, the evolution of the degradation state is treated as a stochastic process so that the RUL pdf can be estimated. In the case of deterministic methods with bounded uncertainties, no distribution is assumed and the exact value of the predicted RUL is assumed to belong to an interval defined by lower and upper bounds. As stochastic approaches, extended Kalman filter (EKF) and particle filter (PF) are investigated. In addition, interval techniques are investigated as an alternative deterministic approach.

The paper is organized as follows. Section II presents the model-based prognosis process. In Section III, two stochastic and a deterministic methods are explained. Section IV describes the simulation results obtained with a crack growth benchmark model. Finally, Section V concludes the paper and presents some directions for future works.

II. PROBLEM STATEMENT

A Prognosis and Health Management (PHM) system provides the ability of fault diagnosis and estimation of RUL. This paper contributes to the work of the PHM community

[1] Elinirina Irena Robinson and Julien Marzat are with ONERA - The French Aerospace Lab, F-91123 Palaiseau, France, {elinirina.robinson@onera.fr, julien.marzat@onera.fr}

[2] Tarek Raïssi is with the CEDRIC-Lab, Conservatoire National des Arts et Metiers, Paris 75141, France, tarek.raissi@cnam.fr

978-1-5090-0657-1/16/$31.00 ©2016 IEEE

by comparing stochastic and deterministic model-based prognosis methodologies using performance metrics. This section presents the steps to calculate the RUL of a system and to evaluate the performance of the prognosis technique used.

A. Degradation model construction

The central idea of model-based prognosis is to use a dynamic mathematical model that describes the evolution of a degradation within a system or a component. The one that is used as an illustration in this paper is a crack growth model.

When a crack forms in a component, its size and its propagation speed must be monitored in order to calculate the RUL of this component. The knowledge of the crack growth governing equation is needed. A widely used one in the case of a fatigue crack growth under cyclic load is the Paris-Erdogan law [9]:

$$\frac{da}{dN} = C(\Delta K)^m \quad , \quad \Delta K = \Delta\sigma\sqrt{\pi a} \tag{1}$$

where a is the crack size, N is the number of cycles, ΔK is the range of stress intensity factor and $\Delta\sigma$ is the stress range. C and m are the unknown model parameters to be estimated. Once a degradation model is available, the goal is now to estimate the state of the degradation and to compute the RUL.

B. Degradation state estimation and RUL calculation

State estimation relates the mathematical model of the degradation with the data from the different sensors to determine the underlying behavior of the system at any time instant. As the degradation model is often nonlinear, suitable state estimation techniques should be used. Furthermore, degradation models involve uncertain parameters and estimation methods have to be robust when the degradation state should be estimated.

Usually, prognosis approaches are based on two parts in the degradation state estimation: (i) the current degradation state estimation and (ii) the future degradation state estimation. During the current state estimation, sensor data are available for a specific observation interval whose size depends on the prediction time t_p. Then, from this instant, the forecasting of the degradation state in the future is realized. The particularity of this step is that state estimation is performed without new measurements. The future state is predicted by taking uncertainties into account until the failure threshold is reached, giving the predicted failure time t_{pf}. Finally, the RUL can be calculated as $RUL(t_p) = t_{pf} - t_p$. Fig. 1 provides a scheme of the process.

As new measurements are collected, predictions are improved via model parameters and degradation state estimates updating. Thus, the uncertainties are reduced over time.

C. Performance evaluation

There is no strict agreement about which appropriate and acceptable set of metrics should be used in prognosis applications. However, it is widely admitted that accuracy and precision indicators are relevant to examine the performance

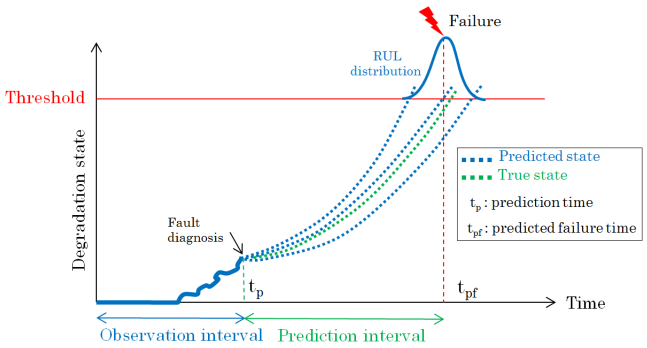

Fig. 1. Prognosis process scheme

of prognosis algorithms [10]. These metrics derive from the prediction error performed at time t_p which is expressed as $e(t_p) = RUL(t_f) - RUL(t_{pf})$ where $RUL(t_f)$ is the ground-truth RUL at the actual time of failure t_f.

- *Accuracy* is a measure of the degree of closeness of predicted failure time t_{pf} to the actual failure time t_f. This metric provides an exponential weight of the errors in RUL predictions over several experiments. The accuracy of a prognosis algorithm at a specific prediction time t_p is defined as [11]:

$$A(t_p) = \frac{1}{N_s}\sum_{n=1}^{N_s} \exp\left(-\frac{|e_n(t_p)|}{RUL(t_f)}\right) \tag{2}$$

  where N_s is the number of experiments and $e_n(t_p)$ is the prediction error of the n^{th} experiment. The range of the accuracy is between 0 and 1, where 1 gives the best accuracy.
- *Precision* is a measure of the narrowness of the interval in which the RUL predictions fall and is expressed as [12]:

$$P(t_p) = \exp\left(-\frac{R}{R_0}\right) \tag{3}$$

  where R is the width of the confidence interval of the prediction given by $R = 2 \times 3\sigma_{RUL}$ where σ_{RUL} is the standard deviation of the RUL pdf. R_0 is a normalizing factor. The precision value varies between 0 and 1, where 1 reflects the highest precision.
- *Timeliness* indicates the relative position of the predicted RUL pdf along the time axis with respect to the occurrence of the actual failure event. There are three cases (Fig. 2): (a) the failure occurs after the predicted failure time t_{pf}, (b) the failure occurs at the same time as the predicted failure time, and finally, (c) the failure occurs earlier than predicted. This last case must be absolutely avoided. To compute the timeliness metric, the following function is used [13]:

$$T = \frac{1}{N}\sum_{n=1}^{N_s} T_n \tag{4}$$

$$T_n = \begin{cases} \exp(-\frac{e_n(t_p)}{R_{min}}) - 1 \text{ , if } e_n(t_p) \leq 0 \\ \exp(\frac{e_n(t_p)}{R_{max}}) - 1 \text{ , if } e_n(t_p) > 0 \end{cases} \tag{5}$$

$[R_{min}, R_{max}]$ represents the interval around the ground-truth RUL (Fig. 2). The values of this timeliness function are in the interval $[0; +\infty]$ and the perfect score for timeliness is 0.

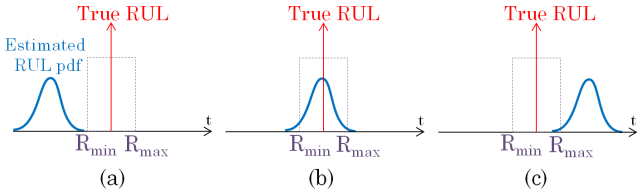

Fig. 2. Timeliness metric

III. DEGRADATION STATE ESTIMATION

In order to solve the dynamic degradation state estimation problem, two mathematical equations are needed: a first one describing the evolution of the degradation state and a second one relating the state and the noisy measurements. The association of these two equations gives the following discrete state-space system:

$$x_k = f(x_{k-1}, \theta_{k-1}, w_k) \tag{6}$$

$$y_k = h(x_k, v_k) \tag{7}$$

where $x \in \mathbb{R}^n$ denotes the state, θ represents the unknown model parameter vector, $y \in \mathbb{R}^p$ is the measured outputs and $k \in \mathbb{N}$ is a discrete time step. The functions f and h describe respectively the nonlinear evolution of the state and the measurements over time. The variables w and v are respectively the process and measurement noises which represent the model and measurements uncertainties. In the context of prognosis, the state to be estimated is the degradation. To this purpose, two stochastic methods, an EKF and a PF are presented in this section, then a deterministic approach using interval techniques is introduced. Determinism implies that noises and disturbances are bounded, whereas in the case of stochastic state estimation, they are modeled in probabilistic terms.

A. Stochastic methods

The objective of stochastic filtering is to estimate the pdf $p(x_k|y_{0:k})$ which gives statistical information about the degradation state x_k, based on the set of all measurements $y_{0:k} = Y_k = \{y_0, \cdots, y_k\}$. The state equation (6) characterizes the state transition pdf $p(x_{k+1}|x_k)$, whereas the measurement equation (7) describes the pdf $p(y_k|x_k)$ which is further related to the measurement noise model. Concerning the parameter vector θ, it is jointly estimated with the state x. In order to handle this estimation problem, the parameters are assumed to be constant over time, *i.e.* $\theta_k = \theta_{k-1}$. This method often provides adequate results when only a few parameters are to be estimated [14].
Given $p(x_0)$, $p(x_{k+1}|x_k)$ and $p(y_k|x_k)$, the recursive Bayesian estimation presented below is used to solve the stochastic filtering problem.

Recursive Bayesian estimation principle

In Bayesian theory, the uncertainties are treated as random variables. Moreover, a recursive filtering approach means that received data can be processed sequentially rather than as a batch so that it is not necessary to store the complete data set nor to reprocess existing data if a new measurement becomes available.
First of all, it is assumed that: (i) the state vector is a first-order Markov process such that $p(x_k|x_{0:k-1}) = p(x_k|x_{k-1})$ and (ii) the observations are independent of the states. Combining these two assumptions and the Bayes rules, the following equation is obtained [15]:

$$p(x_k|Y_k) = \frac{p(y_k|x_k)p(x_k|Y_{k-1})}{p(y_k|Y_{k-1})} \tag{8}$$

The posterior density $p(x_k|Y_k)$ is defined through the combination of three terms:

- The prior density $p(x_k|Y_{k-1})$ which is the prediction density of the state at time k obtained via the Chapman-Kolmogorov equation:

$$p(x_k|Y_{k-1}) = \int p(x_k|x_{k-1})p(x_{k-1}|Y_{k-1})dx_{k-1} \tag{9}$$

  where $p(x_k|x_{k-1})$ is the state transition density defined by the state equation (6);
- The Likelihood density $p(y_k|x_k)$ which is defined by equation (7);
- A normalizing constant $p(y_k|Y_{k-1})$ which depends on the likelihood and the prior such that:

$$p(y_k|Y_{k-1}) = \int p(y_k|x_k)p(x_k|Y_{k-1})dx_k \,. \tag{10}$$

Bayesian filtering handles the computation or approximation of these three terms to deduce the pdf of the degradation state $p(x_k|Y_k)$. It is based on two steps: prediction and update. First, the required pdf $p(x_{k-1}|Y_{k-1})$ is supposed to be available. During the prediction step, using the previous pdf and the system model (6), the prior $p(x_k|Y_{k-1})$ is approximated with equation (9). Then comes the update step at time k when a measurement y_k becomes available, the likelihood pdf $p(y_k|x_k)$ is obtained with the measurement equation (7). Then, the posterior density $p(x_k|Y_k)$ is deduced from equation (8). Based on this Bayesian filtering algorithm, many types of filters have been developed [15]. In this paper, an EKF and a PF are used, since they can handle nonlinear dynamical systems.

Extended Kalman filter

The EKF is an extended version of the original Kalman filter (KF) [16] developed for nonlinear systems. In this filtering approach, the state pdf $p(x_k|Y_k)$ is approximated by a Gaussian distribution. To achieve this, the nonlinear degradation model is linearized around the last predicted degradation state estimate and the conventional KF algorithm is applied to the linearized dynamics. To linearize the nonlinear functions f and h which are assumed to be differentiable, their respective Jacobian matrices F and H are computed at each time step with the predicted degradation state:

$$F_k = \left.\frac{\partial f}{\partial x}\right|_{\hat{x}_{k-1}} \qquad H_k = \left.\frac{\partial h}{\partial x}\right|_{\hat{x}_k} \tag{11}$$

Current degradation state estimation with the EKF:
To solve the prognosis problem, the first step consists in the estimation of the current degradation state, while measurements are available. Hence, the classical EKF algorithm [17] expressed as follows is used:
Prediction step

$$\begin{aligned} \hat{x}_{k|k-1} &= f(\hat{x}_{k-1|k-1}, w_k = 0) \\ P_{k|k-1} &= F_k P_{k-1|k-1} F_k^T + Q_k \end{aligned} \tag{12}$$

Update step

$$\begin{aligned} \hat{y}_k &= y_k - H_k \hat{x}_{k|k-1} \\ K_k &= P_{k|k-1} H_k^T (H_k P_{k|k-1} H_k^T + R_k) \\ P_{k|k} &= (I - K_k H_k) P_{k|k-1} \\ \hat{x}_{k|k} &= \hat{x}_{k|k-1} + K_k \hat{y}_k \end{aligned} \tag{13}$$

where P, Q and R are covariance matrices, respectively of the estimation error, the process noise and the measurements noise. K is the Kalman gain.

Future degradation state estimation with the EKF:
In this part, the previous estimation $\hat{x}_{k_{p-1}|k_{p-1}}$ is used as the initial degradation state while the prediction step remains the same. The update step is changed because the innovation term $\hat{y}_k = y_k - H_k \hat{x}_{k|k-1}$ is no more available. Instead, the degradation state at step $k \in \{k_p, \cdots, k_{pf}\}$ is updated with the state transition model (6) using the standard deviation of the previous degradation state to approximate the distribution of the noise w. The algorithm for the estimation of the future degradation step from step k_p to step k_{pf} is given by the following equations:
Prediction step

$$\begin{aligned} \hat{x}_{k|k-1} &= f(\hat{x}_{k-1|k-1}, w_k = 0) \\ P_{k|k-1} &= F_k P_{k-1|k-1} F_k^T + Q \end{aligned} \tag{14}$$

Update step

$$\begin{aligned} P_{k|k} &= P_{k|k-1} \\ \hat{w}_k &\sim N(0, P_{k|k}) \\ \hat{x}_{k|k} &= f(\hat{x}_{k|k-1}, \hat{w}_k) \end{aligned} \tag{15}$$

The EKF generally provides satisfying results and is easy to implement. But as the desired state pdf is approximated by a Gaussian distribution, it may have significant deviation from the true distribution causing divergence in the case where the degradation model is highly nonlinear. In order to deal with more complex degradation models, particle filters can be used.

Particle filter

In the PF approach, the state pdf at time instant k is approximated by a set of N_p particles $\{x_k^i\}_{i=1}^{N_p}$ representing points in the unknown state space, and a set of associated weights $\{\omega_k^i\}_{i=1}^{N_p}$ denoting discrete probability masses:

$$p(x_k|Y_k) \approx \sum_{i=1}^{N_p} \omega_k^i \delta(x_k - x_k^i) \text{ with } \sum_{i=1}^{N_p} \omega_k^i = 1 \tag{16}$$

Ideally to represent samples, the particles should be drawn from the pdf $p(x_k|Y_k)$. However, as it is often impossible, an alternative easy-to-sample proposal distribution $q(x_k|Y_k)$ is used instead. Usually, the importance density function is set equal to the *a priori* state pdf, which means $q_k(x_k|x_{k-1}) = p(x_k|x_{k-1})$.

Current degradation state estimation with the PF:
There exist several PF algorithms (see [18]). One of the most used is the sequential importance resampling (SIR) particle filter. It is based on three main steps which are prediction, update and re-sampling:
Initialization

- Draw particles $x_0^i \sim p(x_0)$
- Compute the initial weights $\omega_k^i = \frac{1}{N_p}$

Prediction step

- Simulate the state model (6) to generate a new set of N_p particles $x_k^{i=1:N_p}$ which are realizations of the predicted pdf $p(x_k|Y_{k-1})$.

Update step

- Each sampled particle is assigned a weight based on the likelihood $p(y_k|x_k)$:

$$\omega_k^i = \omega_{k-1}^i p(y_k|x_{k-1}^i) = \omega_{k-1}^i \frac{p(y_k|x_k^i) p(x_k^i|x_{k-1}^i)}{p(x_k^i|x_{k-1}^i, y_k)} \tag{17}$$

- Normalize the weights:

$$\omega_k^i = \omega_k^i (\sum_{i=1}^{N_p} \omega_k^i)^{-1} \tag{18}$$

Re-sampling

- Degeneracy problem: the weight variance increases and after a few iterations all but one particle have a negligible weight [19]. Particles with small weights are eliminated so that the computational efforts are concentrated in those having large ones.
- Re-sampling condition: if the effective sample size N_{eff} is under some threshold N_{th}, a re-sampling procedure is

done. An estimate of N_{eff} is

$$\tilde{N}_{eff} = (\sum_{i=1}^{N_p} (\omega_k^i)^2)^{-1} \quad (19)$$

- Using the inverse cumulative distribution function (CDF) method [18] and the current set $\{x_k\}_{i=1}^{N_p}$, a new set $\{\tilde{x}_k\}_{i=1}^{N_p}$ is drawn to replace the current one. Finally, with $\tilde{\omega}_k^i = N_p^{-1}$, the state is given by:

$$\hat{x}_k^i = \sum_{i=1}^{N_p} \tilde{\omega}_k^i \tilde{x}_k^i \quad (20)$$

The prediction and update steps form a single iteration and are recursively applied at each time k, whereas the re-sampling step execution depends on the value of N_{eff}.

Future degradation state estimation with the PF:
To apply the PF algorithm and to obtain the degradation state pdf, the weight of each particle should be updated at every step. However, these weights depend on the acquisition of new measurements. To overcome this difficulty, the state is propagated only using the state model (6) while the current particle weights are propagated in time without any changes. In other words, only the prediction step is repeated until the threshold is reached.

B. Set-membership framework

In this subsection, a set-membership methodology is proposed based on a guaranteed estimation and prediction of models described by (6), (7) where only the bounds of the noises and disturbances are available without any additional stochastic assumption. Disturbances w and noises v satisfy $|v_k| \leq V$, $|w_k| \leq W$ for some positive bounds V and W. Since w and v belong to intervals, the parameter vector θ and the degradation state x can not take single values but they also belong to some compact domains. The proposed set-membership methodology is based on two steps:

- The available measurements over the time interval $[t_0, t_p]$ are used to estimate the feasible domain of the parameter vector θ given by

$$\Theta = \left\{ \begin{array}{l} \theta \in \mathbb{R}^q \mid x_k = f(x_{k-1}, \theta, w_k), \\ h(x_k, v_k) \in [y_k^m - V, y_k^m + V], \\ \forall k \in [0, t_p], \forall x_{k-1} \in [x_{k-1}], \forall |w_k| \leq W \end{array} \right\} \quad (21)$$

  where y_k^m are the measurements and $[y_k^m - V, y_k^m + V]$ is the domain of the output taking into account the noises.
- The estimated feasible parameter domain is used to predict the degradation behavior for $t_k > t_p$ in order to estimate the remaining useful life of the system.

The degradation models used in this paper are nonlinear and the estimation and prediction steps are based on interval tools to take into account the bounded uncertainties.

Interval techniques

Interval analysis techniques represent a powerful tool to tackle uncertainty propagation without any stochastic assumption. Indeed, the evaluation of the whole set of possible model outputs could be performed using only one interval evaluation. A real interval $[a] = [\underline{a}, \overline{a}]$ is a connected and closed subset of $\mathbb{R}$. The set of all real intervals of $\mathbb{R}$ is denoted by $\mathbb{IR}$. Real arithmetic operations are extended to intervals (see [20]). Consider an operation $\circ \in \{+; -; *; /\}$ and $[a]$, $[b]$ two intervals, then:

$$[a] \circ [b] = \{x \circ y \mid x \in [a], y \in [b]\}$$

The width of an interval $[a]$ is defined by $w[a] = \overline{a} - \underline{a}$ and its midpoint by $mid[a] = (\underline{a} + \overline{a})/2$. The midpoint represents a point estimation of a variable and the radius is the uncertainty.
Let $f : \mathbb{R}^n \to \mathbb{R}^m$; the range of the function f over an interval vector (called also a box) $[x]$ is given by:

$$f([x]) = \{f(x) \mid x \in [x]\} \quad (22)$$

An interval function $[f] : \mathbb{IR}^n \to \mathbb{IR}^m$ is an inclusion function for f if:

$$\forall [x] \in \mathbb{IR}^n, \quad f([x]) \subseteq [f]([x]) \quad (23)$$

An inclusion function of f could be obtained by replacing each occurrence of a real variable by its corresponding interval and by replacing each standard function by its interval evaluation. Such a function is called the natural inclusion function.
A constraint satisfaction problem (CSP) is defined by a set of n variables $x = x^1, x^2, \ldots, x^n$ and a set of m constraints $\mathscr{C}_1, \mathscr{C}_2, \ldots, \mathscr{C}_m$. Each variable x^i has an initial nonempty domain $\mathscr{D}_i$ of possible values. Each constraint $\mathscr{C}_i$ involves a subset of the variables and specifies the possible combinations of values for such subset. A state of the problem is defined by an assignment of values to some or all of the variables, $x^i = v_i, \ldots, x^j = v_j$. An assignment that does not violate any constraint is called a consistent assignment. A complete assignment is one in which every variable is mentioned, and a solution to a CSP is a complete assignment that satisfies all the constraints. In contrast to conventional techniques, interval methods do not suffer from local convergence and the computed set is guaranteed to contain the global solutions. In addition, an empty set is returned if the CSP has no solution in the initial searching domain. The goal of propagation techniques is to reduce as much as possible the domains for the variables without losing any solution. The most known approach is based on the Waltz filtering algorithm [21] which has initially been proposed to reduce the combinatory associated with line labeling of three-dimensional scenes. It has proved its effectiveness in solving some control problems such as identification, filtering and robust control [22].
A contractor associated to a set X is an operator $\mathscr{C}$ which associates to a box $[x] \in \mathbb{R}^n$ another box $C([x]) \in \mathbb{R}^n$ such that the two following properties are always satisfied [22]:

- $C([x]) \subset [x]$ (contractance property)
- $C([x]) \cap X = [x] \cap X$ (completeness property)

Damage estimation

In the sequel, we propose to use interval techniques for estimating the parameters of system (21). Thus, the following CSP is formulated:

$$\mathscr{C}: \begin{cases} \mathscr{C}_k : x_k = f(x_{k-1}, \theta, w_k),\ y_k = h(x_k, u_k),\ k = 0, \ldots t_p \\ \theta \in [\underline{\theta}, \overline{\theta}] \end{cases} \tag{24}$$

In the following, an outer and inner approximations of the solution set defined by (21) are characterized using the algorithm contractor based on the Waltz filtering algorithm. This methodology allows one to compute two sets Θ_{int} and Θ_{ext} of intervals satisfying:

$$\Theta_{int} \subseteq \Theta \subseteq \Theta_{ext} \tag{25}$$

In the second step, the degradation prediction is computed by the means of the natural inclusion function of the state equation (6), where $[\theta]$ is computed through a projection of Θ_{ext} on the axes θ_i. This inclusion function is given by

$$[x_k] = [f]([x_{k-1}], [\theta], [-W, W]) \tag{26}$$

where $[-W, W]$ is the feasible domain of the disturbances w. A consistency check of the predicted intervals $[x_k]$ ($k > t_p$) and the degradation threshold is used to estimate the remaining residual life as shown in Fig. 5. Due to uncertainty propagation, it is not possible to compute a reliable point estimation of the RUL. In the following, we propose to define lower and upper bounds of the RUL (*i.e.* $t_{RUL} \in [\underline{t}_{RUL}, \overline{t}_{RUL}]$) defined by:

$$\begin{cases} \underline{t}_{RUL} = \underline{t}_{pf} - t_p \\ \overline{t}_{RUL} = \overline{t}_{pf} - t_p \end{cases} \tag{27}$$

Similarly to the stochastic case where the RUL is characterized by a probability density function, in the set-membership context, the RUL can be considered as a random variable with an uniform pdf within the bounds $[\underline{t}_{RUL}, \overline{t}_{RUL}]$.

IV. NUMERICAL RESULTS

In this section, the three model-based prognosis methods presented are applied to estimate the evolution of a nonlinear fatigue crack growth process. The simulation results are shown and compared using the performance metrics given in Section II-C. With dN sufficiently small, the Paris model (1) can be discretized to give:

$$a_k = e^{C_k} (\Delta\sigma \sqrt{\pi a_{k-1}})^{m_k} dN + a_{k-1} \tag{28}$$

Therefore, the augmented state vector to be estimated is:

$$x_k = [a_k, C_k, m_k]^T \tag{29}$$

In this model, $\log(C)$ is used because C has a very small value. The true crack size data is generated using the values given in Table I and measurements are obtained by adding a uniform noise distributed in the interval $[-0.002, 0.002]$. Concerning the stochastic methods, Table II gathers the variance of the process noise, of the measurements noise, and of the parameters m and C.

TABLE I

SIMULATION PARAMETERS

$\Delta\sigma$	dN	a_{true}	$log(C_{true})$	m_{true}
78	50	0.01	-22.33	3.5

TABLE II

PARAMETERS OF THE EKF AND PF ALGORITHMS

σ_C^2	σ_m^2	σ_w^2	σ_v^2
10^{-2}	10^{-3}	10^{-8}	10^{-10}

The choice of these hyper-parameters has been done experimentally after some simulations. It was noticed that when decreasing σ_v^2, the RUL calculated with the EKF and the PF were overestimatd and when increasing it the predicted RUL were underestimated. Moreover, the three algorithms are very sensitive to the initial value of the parameter m as it is an exponent.

The experiments are performed assuming that the true values of the parameters are unknown, however their variation ranges are known [23]. 24 measurements were generated every 50 cycles, from cycle 0 to cycle 1200, which is the prediction time t_p. From this time instant, the estimation of the degradation state in the future without new measurements was realized until the threshold fixed at 0.0463 is reached (according to [24]). In order to evaluate the performance of the algorithms, 100 experiments have been simulated, and the value of the performance metrics parameters are $[R_{min}, R_{max}] = R_0 = 100$. The simulation results are depicted from Fig. 3 to Fig. 8.

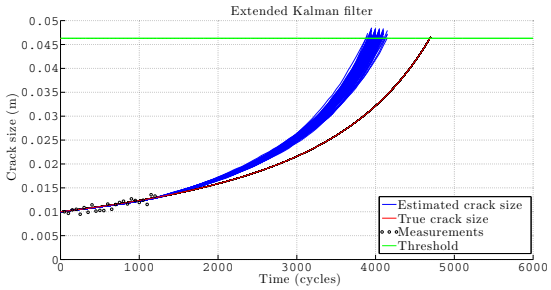

Fig. 3. Results of 100 experiments obtained with an EKF

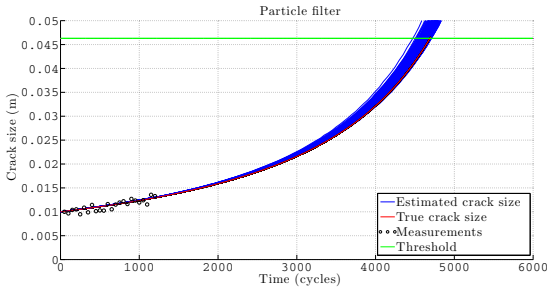

Fig. 4. Results of 100 experiments obtained with a PF

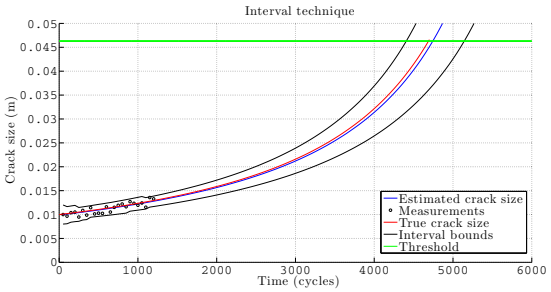

Fig. 5. Results obtained with interval technique

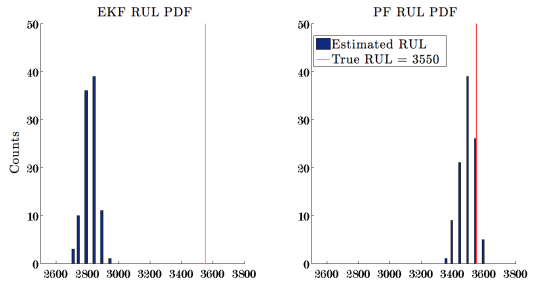

Fig. 6. RUL pdf for the EKF and the PF

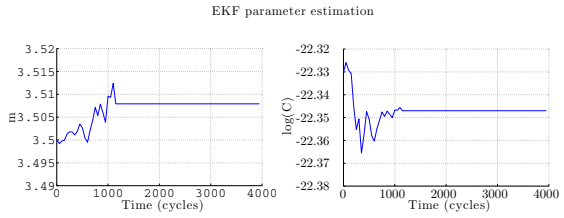

Fig. 7. Estimation of the parameters m and C with the EKF

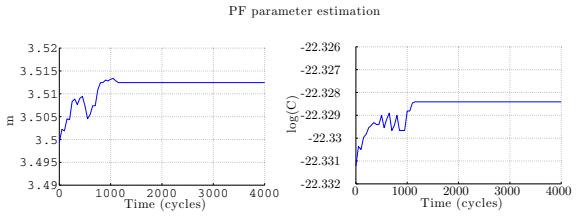

Fig. 8. Estimation of the parameters m and C with the PF

TABLE III

PERFORMANCE EVALUATION RESULTS

Method	Accuracy	Precision	Timeliness
EKF	0.8151	0.7501	1.8277
PF	0.9842	0.7283	0.0873
Intervals	0.9983	0.4790	0.6927

The simulations show that the EKF based method is less performant than the PF in terms of accuracy and timeliness. Indeed, because of the nonlinear dynamics, the different pdfs involved in the nonlinear Bayesian filtering problem are not Gaussian while the EKF algorithm assumes them as Gaussian, which may lead to the divergence of the filter. The lower performance of the EKF has already been reported in [25]. However, it was not quantified precisely with metrics. The deterministic method based on interval techniques gets the best results concerning accuracy. However, its precision is smaller than the particle filter. In terms of timeliness, the last two algorithms are in the case where the predicted RUL pdf is around the ground-truth RUL. Moreover, the PF approach requires a more complex implementation and has to propagate the entire state pdf at each step which tends to increase the computational time compared to the two other algorithms. This work also shows the interest of using the metrics presented in Section II-C for evaluating prognosis peformance.

V. CONCLUSIONS

Deterministic and stochastic model-based prognosis approaches with uncertainty propagation have been compared using different performance metrics. Both kinds of methods are able to generate a pdf or an interval that encapsulate the different uncertainties associated to the RUL prediction. It was shown that the PF method outperforms the EKF one. Then it was observed that the accuracy of the interval method is higher, but the PF approach results in a narrower RUL pdf. However, the interval generated in the deterministic method tends to shrink as more measurements are available. Therefore one can conclude that the choice between the two algorithms (namely PF or interval technique) can be driven by user requirements and available resources considerations. For example, if a low computation time is needed and if the evolution of the degradation state is rather slow, the deterministic method based on interval techniques can be used as the precision improves over time. In the case where prediction horizons are smaller, the PF approach should be preferred.

In further works more realistic data will be used, and other more complex degradation models will be considered, such as a turbofan degradation model. Finally, the uncertainty management can be highly enhanced with a sensitivity analysis which consists in quantifying the influence of each source of uncertainty to identify the most significant input variables or model parameters. Moreover, a combination of stochastic and deterministic methods is also a conceivable solution to bridge the gap between the two algorithms and to take advantage of possible complementarities.

REFERENCES

[1] H. Liu, J. Yu, P. Zhang, and X. Li, "A review on fault prognostics in integrated health management," *Proceedings of 9th International Conference on Electronic Measurement and Instruments*, pp. 4267–4270, 2009.

[2] T. Biagetti, "Automatic diagnostics and prognostics of energy conversion processes via knowledge-based systems," *Energy*, vol. 29, no. 12-15, pp. 2553–2572, 2004.

[3] X.-S. Si, W. Wang, C.-H. Hu, and D.-H. Zhou, "Remaining useful life estimation A review on the statistical data driven approaches," *European Journal of Operational Research*, vol. 213, no. 1, pp. 1–14, 2011.

[4] J. Luo, M. Namburu, K. Pattipati, L. Qiao, M. Kawamoto, and S. Chigusa, "Model-based prognostic techniques [maintenance applications]," in *Proceedings of IEEE AUTOTESTCON Conference*, pp. 330–340, 2003.

[5] C. S. Byington, M. Watson, D. Edwards, and P. Stoelting, "A model-based approach to prognostics and health management for flight control actuators," vol. 6, pp. 3551–3562, 2004.

[6] B. Saha and K. Goebel, "Uncertainty Management for Diagnostics and Prognostics of Batteries using Bayesian Techniques," pp. 1–8, 2008.

[7] P. Baraldi, F. Mangili, and E. Zio, "Investigation of uncertainty treatment capability of model-based and data-driven prognostic methods using simulated data," *Reliability Engineering and System Safety*, vol. 112, pp. 94–108, 2013.

[8] S. Sankararaman and K. Goebel, "Remaining useful life estimation in prognosis: An uncertainty propagation problem," *AIAA Infotech at Aerospace (I at A) Conference*, pp. 1–8, 2013.

[9] P. Paris and F. Erdogan, "A critical analysis of crack propagation laws," *Journal of basic engineering*, vol. 85, no. 4, pp. 528–533, 1963.

[10] D. Edwards, M. E. Orchard, L. Tang, K. Goebel, and G. Vachtsevanos, "Impact of input uncertainty on failure prognostic algorithms: Extending the remaining useful life of nonlinear systems," tech. rep., DTIC Document, 2010.

[11] G. Vachtsevanos, F. Lewis, M. Roemer, A. Hess, and B. Wu, *Intelligent fault diagnosis and prognosis for engineering systems*, vol. 13. 2006.

[12] S. Butler, *Prognostic Algorithms for Condition Monitoring and Remaining Useful Life Estimation*. PhD thesis, National University of Ireland, Maynooth, 2012.

[13] A. Saxena, J. Celaya, E. Balaban, K. Goebel, B. Saha, S. Saha, and M. Schwabacher, "Metrics for evaluating performance of prognostic techniques," in *International Conference on Prognostics and Health Management (PHM)*, pp. 1–17, IEEE, 2008.

[14] J. Ching, J. L. Beck, and K. A. Porter, "Bayesian state and parameter estimation of uncertain dynamical systems," *Probabilistic engineering mechanics*, vol. 21, no. 1, pp. 81–96, 2006.

[15] Z. Chen, "Bayesian filtering: From kalman filters to particle filters, and beyond," *Statistics*, vol. 182, no. 1, pp. 1–69, 2003.

[16] R. E. Kalman and R. S. Bucy, "New results in linear filtering and prediction theory," *Journal of basic engineering*, vol. 83, no. 1, pp. 95–108, 1961.

[17] A. Jazwinski, *Stochastic Processes and Filtering Theory*. Dover Books on Electrical Engineering Series, Dover Publications, 2007.

[18] M. S. Arulampalam, S. Maskell, N. Gordon, and T. Clapp, "A tutorial on particle filters for online nonlinear/non-Gaussian Bayesian tracking," *IEEE Transactions on Signal Processing*, vol. 50, no. 2, pp. 174–188, 2002.

[19] F. Daum, "Nonlinear filters: beyond the Kalman filter," *Aerospace and Electronic Systems Magazine*, vol. 20, no. 8, pp. 57–69, 2005.

[20] R. Moore, *Interval analysis*. Englewood Cliffs: NJ : Prentice-Hall, 1966.

[21] D. L. Waltz, "Generating semantic descriptions from drawings of scenes with shadows," in *The psychology of computer vision*, pp. 19–91, New York: McGraw-Hill, 1975.

[22] L. Jaulin, M. Kieffer, O. Didrit, and E. Walter, *Applied interval analysis*. London: Springer, 2001.

[23] J. Bourinet and M. Lemaire, "Form sensitivities to correlation: application to fatigue crack propagation based on virkler data," in *Proc. 4th Int. ASRANet Colloquium*, 2008.

[24] D. An, J. H. Choi, and N. H. Kim, "Prognostics 101: A tutorial for particle filter-based prognostics algorithm using Matlab," *Reliability Engineering and System Safety*, vol. 115, pp. 161–169, 2013.

[25] B. Saha, K. Goebel, and J. Christophersen, "Comparison of prognostic algorithms for estimating remaining useful life of batteries," *Transactions of the Institute of Measurement and Control*, 2009.